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INFLUENCE OF COUPLE STRESSES ON THE PROPAGATION OF ELASTIC WAVES IN A MICROPOLAR CUBICALLY ANISOTROPIC MEDIUM

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Expressions for the phase and radial velocities of propagation of three-dimensional fronts of quasilongitudinal and quasitransverse elastic waves in a micropolar cubically anisotropic medium have been obtained. An analysis of the influence of the micropolar elastic constant on the dependence of the wave velocities on the angle of inclination of the normal to the wave surface has been made.

The micropolar (nonsymmetric) theory of elasticity of a continuum was developed with the aim of eliminating the disagreement between theory and experiment in problems of high-frequency oscillations and describing phenomena occurring in anisotropic media in traversal of elastic waves, etc., with a sufficient degree of accuracy. The theoretical foundations of the nonsymmetric theory of elasticity of an isotropic medium have been reflected in fundamental monographs [1, 2]. The results of experimental investigations on detection of couple-stress effects have been presented in [3, 4]. The micropolar theory of elasticity of an anisotropic medium and an analysis of wave motions in such media have been the focus of [5–7]. Below, we discuss the influence of one micropolar elastic constant on the velocities of propagation of quasilongitudinal and quasitransverse waves in cubically anisotropic materials characterized by the presence of no lower than threefold axes of symmetry [8]. Media with cubic symmetry include such widespread metals and minerals as brass, lead, silver, nickel, table salt, ice, and others.

The resolving system of dynamic equilibrium equations will be represented in the following form [9]:

$$A_3\Delta u_j + A\partial_j^2 u_j + (A_2 + A_4)\,\partial_j \sum_{k=1}^{3} \partial_k u_k + (A_3 - A_4) \sum_{k,m=1}^{3} \varepsilon_{jkm}\varphi_{m,k} + \rho f_j = \rho \ddot{u}_j\,,$$
$$B_3\Delta\varphi_j + B\partial_j^2\varphi_j + (B_2 + B_4)\,\partial_j \sum_{k=1}^{3} \partial_k\varphi_k + (A_3 - A_4) \sum_{k,m=1}^{3} \varepsilon_{jkm}\partial_k u_m - 2\,(A_3 - A_4)\,\varphi_j + \rho l_j = J\rho\ddot{\varphi}_j\,. \qquad (1)$$

The characteristic equation [10] for system (1) breaks down into two uncoupled equations:

$$(A_3\tau_1 - \rho p_0^2)^3 + \tau_1\,(A_1 - A_3)\,(A_3\tau_1 - \rho p_0^2)^2 + A\,(A + 2\,(A_2 + A_4)) \times$$
$$\times (A_3\tau_1 - \rho p_0^2)\,\tau_2 + A^2\,(A + 3\,(A_2 + A_4))\,\tau_3 = 0\,, \qquad (2)$$

$$(B_3\tau_1 - J\rho p_0^2)^3 + \tau_1\,(B_1 - B_3)\,(B_3\tau_1 - J\rho p_0^2)^2 + B\,(B + 2\,(B_2 + B_4)) \times$$
$$\times (B_3\tau_1 - J\rho p_0^2)\,\tau_2 + B^2\,(B + 3\,(B_2 + B_4))\,\tau_3 = 0\,. \qquad (3)$$

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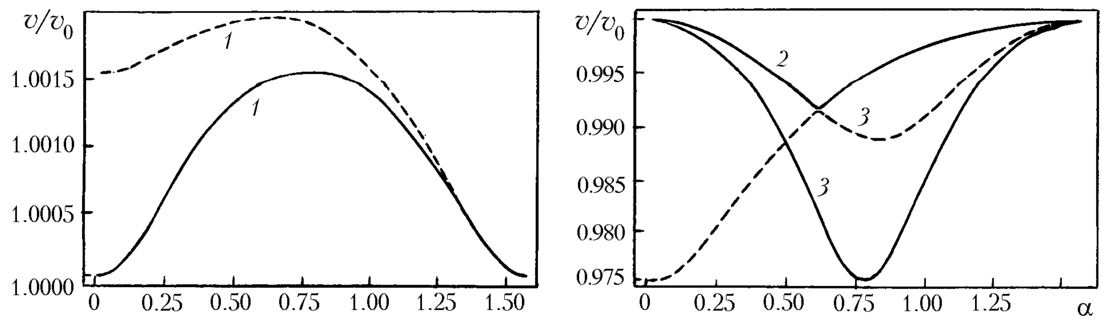

Fig. 1. Dependences of v_j/v_{0j} on the angle α of inclination of the normal to the characteristic surface: 1 and 3) $v_1(\alpha)/v_{01}(\alpha)$ and $v_3(\alpha)/v_{03}(\alpha)$ in the $x_1 = 0$ (solid curves) and $x_1' = 0$ (dashed curves) planes; 2) $v_2(\alpha)/v_{02}(\alpha)$ in the $x_1' = 0$ plane.

Next we restrict ourselves to an analysis of Eq. (2), which can be written as

$$(\tau_1 - p_0^2/c^2)^3 + \tau_1 (a-1)(\tau_1 - p_0^2/c^2)^2 + (a-b-\varepsilon-1)(1+b+\varepsilon-1)\times$$

$$\times(\tau_1 - p_0^2/c^2)\,\tau_2 + (a-b-\varepsilon-1)^2 (a+2b+2\varepsilon-1)\,\tau_3 = 0\,. \tag{4}$$

Taking into account that $V = -p_0/\sqrt{\tau_1}$ and $n_j = p_j\sqrt{\tau_1}$, from (4) we will have

$$(1-v^2)^3 + (a-1)(1-v^2)^2 + (a-b-\varepsilon-1)(a+b+\varepsilon-1)\times$$

$$\times(1-v^2)\,\hat{\tau}_2 + (a-b-\varepsilon-1)^2(a+2b+2\varepsilon-1)\,\hat{\tau}_3 = 0$$

or

$$q_0 v^6 + \hat{q}_1 v^4 + \hat{q}_2 v^2 + \hat{q}_3 = 0\,. \tag{5}$$

In Eq. (5), we have introduced the following notation for the coefficients: $q_0 = -1$, $q_1 = (2+a)\tau_1$, $\hat{q}_1 = 2+a$, $\hat{q}_2 = -(1+2a) - (a-b-\varepsilon-1)(a+b+\varepsilon-1)\hat{\tau}_2$, and $\hat{q}_3 = a + (a-b-\varepsilon-1)(a+b+\varepsilon-1)\hat{\tau}_2 + (a-b-\varepsilon-1)^2(a-1+2b+ 2\varepsilon)\hat{\tau}_3$.

Expressions for the velocities of propagation of elastic waves in the direction of the normal to the wave surface will be represented in the form

$$v_j = \sqrt{\frac{2+a}{3} + 2\sqrt{-\frac{\hat{p}}{3}}\cos\left(\frac{\hat{\Lambda}_j + 2\pi(4-j)}{3}\right)}\,, \quad \hat{\Lambda}_j = \arccos\left(-\frac{\hat{q}}{2}\sqrt{-\left(\frac{3}{\hat{p}}\right)^3}\right), \tag{6}$$

where $\hat{p} = -\dfrac{\hat{q}_1^2}{3q_0^2} + \dfrac{\hat{q}_2}{q_0}$; $\hat{q} = \dfrac{2\hat{q}_1^3}{27q_0^3} - \dfrac{\hat{q}_1\hat{q}_2}{3q_0^2} + \dfrac{\hat{q}_3}{q_0}$.

A comparative analysis of the inverse-velocity surfaces $1/v_j$, which was performed for different cubically anisotropic materials and ε values from the range 1.01–1.025, has shown that $v_1 > v_2 \geq v_3$. Based on the results of [2], it can be inferred that it is a quasilongitudinal wave that propagates with a velocity v_1, whereas quasitransverse waves propagate with velocities v_2 and v_3.

Let us consider the dependences of v_j on the angle α of inclination of the normal to the characteristic surface for the ε ratios equal to 1.25 in one coordinate plane and in the plane $x_1' = 0$ making an angle π/4 with the coordinate planes $x_1 0 x_3$ and $x_2 0 x_3$. Figure 1 gives the dependences of the ratios $v_j(\alpha)/v_{0j}(\alpha)$ in the $x_1 = 0$ and $x_1' = 0$ plane of a

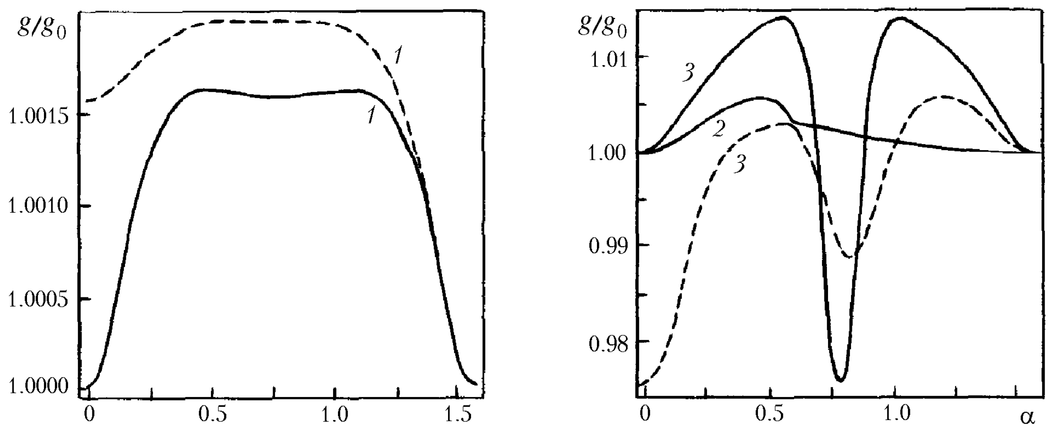

Fig. 2. Dependences of g_j/g_{0j} on the angle of inclination of the normal to the characteristic surface: 1 and 3) $g_1(\alpha)/g_{01}(\alpha)$ and $g_3(\alpha)/g_{03}(\alpha)$ in the $x_1 = 0$ (solid curves) and $x_1' = 0$ (dashed curves) planes; 2) $g_2(\alpha)/g_{02}(\alpha)$ in the $x_1' = 0$ plane.

cubically anisotropic material ($v_{0j}(\alpha)$ is the dependence of the velocity of propagation of an elastic wave for $\varepsilon = 1$). The elastic properties of the medium are characterized by the constants $a = 3.23611$ and $b = 2.72222$ [11].

From Fig. 1, it is clear that the maximum change in the velocities v_1 and v_3 as compared to the velocities v_{01} and v_{03} determined within the framework of the classical elasticity theory is observed in the plane $x_1 = 0$ at $\alpha = \pi/4$; this difference is more pronounced for the quasitransverse wave (≈2%). As ε increases further, the velocity of the quasilongitudinal wave increases, whereas the velocity of the quasitransverse wave decreases as compared to v_1 and v_3. In the case where $\varepsilon < 1$ we have $v_1(\alpha)/v_{01}(\alpha) \le 1$ and $v_3(\alpha)/v_{03}(\alpha) \ge 1$ for any angles of inclination of the normal to the characteristic surface. The velocity of propagation of the quasitransverse wave v_2 in the coordinate plane is unaffected by the micropolar elastic constant $\varepsilon = A_4/A_3$. In the $x_1' = 0$ plane, couple stresses lead to an increase in the velocities of three elastic waves, including the velocity of the quasitransverse wave v_2. Its largest quantitative change in this plane amounts to ≈0.8% at $\alpha = 0.6$. The same value of the angle of inclination of the normal in the $x_1' = 0$ plane corresponds to the maximum change in the velocity of the quasilongitudinal wave (excess of ≈0.2% as compared to v_{01}). It is noteworthy that the values of the ratios v_j/v_{0j} in the $x_1' = 0$ plane at $\alpha = 0$ and in the $x_1 = 0$ plane at $\alpha = \pi/4$ exactly coincide.

The modulus of the radial velocity of propagation of an elastic wave (modulus of the velocity of propagation of wave energy) is determined by the following expression [12]:

$$G = \sqrt{\sum_{i=1}^{3} \left(\frac{\partial p_0}{\partial p_i}\right)^2} \,. \tag{7}$$

We note that the radial velocity is numerically equal to the path traversed by wave energy in this direction per unit time [8].

We express p_0 from Eq. (4):

$$\frac{p_0^{(j)}}{c} = \sqrt{\frac{(2+a)\,\tau_1}{3} + 2\sqrt{-\frac{p}{3}}\cos\left(\frac{\Lambda_j + 2\pi\,(4-j)}{3}\right)} \,, \tag{8}$$

where $\Lambda_j = \arccos\left(-\frac{q}{2}\sqrt{-\left(\frac{3}{p}\right)^3}\right)$, $p = \frac{q_1^2}{3q_0^2} + \frac{q_2}{q_0}$, $q = \frac{2q_1^3}{27q_0^3} - \frac{q_1 q_2}{3q_0^2} + \frac{q_3}{q_0}$, $q_1 = (2+a)\tau_1$; $q_2 = -(1+2a)\tau_1^2 - (a-b-\varepsilon-1)(a+b+\varepsilon-1)\tau_2$; $q_3 = a\tau_1^3 - (a-b-\varepsilon-1)(a+b+\varepsilon-1)\tau_1^2\tau_2 + (a-b-\varepsilon-1)^2(a-1+2b+2\varepsilon)\tau_3$, the superscript j determines the type of elastic wave.

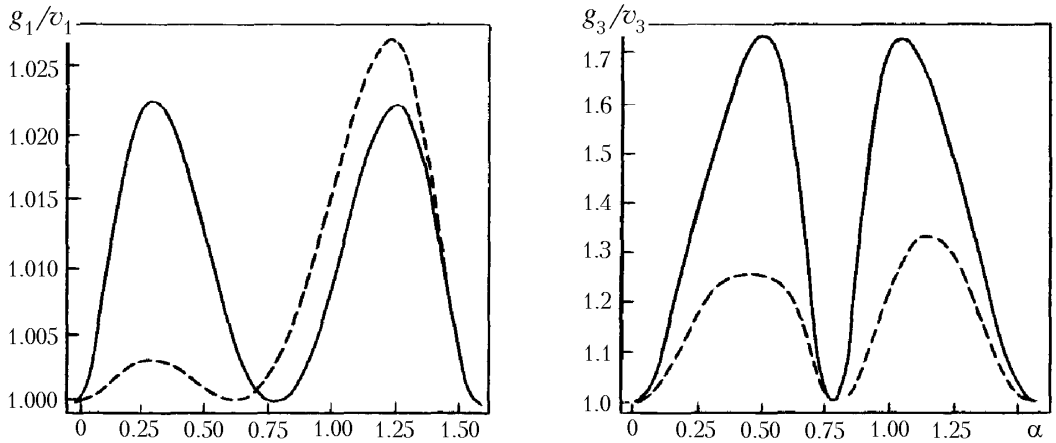

Fig. 3. Dependences of the velocity ratios g_1/v_1 and g_3/v_3 on the angle α of inclination of the normal to the characteristic surface in the $x_1 = 0$ (solid curves) and $x_1' = 0$ (dashed curves) planes of a cubically anisotropic body.

We find the partial derivatives of p_0 from (8) with respect to the parameters p_i:

$$\frac{1}{c}\frac{\partial p_0^{(j)}}{\partial p_i} = \frac{1}{v_j}\left(\frac{1}{2\sqrt{-3\hat{p}}}\left(\frac{2\hat{q}_1 q_{1i}}{3q_0^2} - \frac{q_{2i}}{q_0}\right)\cos\left(\frac{\hat{\Lambda} + 2\pi(4-j)}{3}\right) - \frac{1}{3}\sqrt{-\frac{\hat{p}}{3}}\,\times\right.$$

$$\times\sin\left(\frac{\hat{\Lambda} + 2\pi(4-j)}{3}\right)\sqrt{\frac{\hat{p}^3}{4\hat{p}^3 + 27\hat{q}^2}}\left(\left(\frac{2\hat{q}_1^2 q_{1i}}{9q_0^3} - \frac{\hat{q}_2 q_{1i} + \hat{q}_1 q_{2i}}{3q_0^2} + \frac{q_{3i}}{q_0}\right)\times\right.$$

$$\left.\left.\times\sqrt{\left(-\frac{3}{\hat{p}}\right)^3} - \frac{9\hat{q}\sqrt{3}}{2}\sqrt{\left(-\frac{1}{\hat{p}}\right)^5}\left(\frac{2\hat{q}_1 q_{1i}}{3q_0^2} - \frac{q_{2i}}{q_0}\right)\right)\right), \quad i, j = \overline{1, 3}\,. \tag{9}$$

The coefficients q_{ki}, $k = \overline{1, 3}$, with account for $p_j = n_j\sqrt{\tau_1}$ will be represented in the form

$$q_{1i} = (2 + a)\,\tau_{1i}\,, \quad q_{2i} = (a - b - \varepsilon - 1)\,(a + b + \varepsilon - 1)\,\tau_{2i} - 2\,(1 + 2a)\,\tau_{1i}\,,$$

$$q_{3i} = 3a\tau_{1i} + (a - b - \varepsilon - 1)\,(a + b + \varepsilon - 1)\,(\tau_{1i}\hat{\tau}_2 + \hat{\tau}_1\tau_{2i}) + (a - b - \varepsilon - 1)^2\,(a - 1 + 2b + 2\varepsilon)\,\tau_{3i}\,.$$

Substituting (9) into (7), we will have the expressions for the dimensionless radial velocities of propagation of elastic waves $g_j = G_j/c$. Figure 2 gives the dependences of the ratios g_j/g_{0j} on the angle α in the $x_1 = 0$ and $x_1' = 0$ planes of a cubically anisotropic material for $\varepsilon = 1.025$. The elastic properties of the material are characterized by the constants $a = 3.23611$ and $b = 2.72222$.

Figure 2 shows that the radial velocity of propagation of the quasilongitudinal wave with allowance for the micropolar effects in the $x_1 = 0$ and $x_1' = 0$ planes exceeds the velocity of the elastic wave irrespective of the angle of inclination of the normal to the characteristic surface. In the $x_1' = 0$ plane, the influence of couple stresses on the velocity of this wave is more significant than that in the $x_1 = 0$ plane. The velocity g_3 of propagation of the quasitransverse wave in the two planes in question as a function of the angle of inclination of the normal to the wave surface can be higher or lower than the velocity g_{03}; the influence of the micropolar elastic constant in the $x_1' = 0$ plane is more substantial than that in the coordinate plane. In the $x_1' = 0$ plane, the micropolar constant affects the velocity g_2 of propagation of another quasitransverse wave, too (in the $x_1 = 0$, the ratio g_2/g_{02} takes on a value of unity irrespective the angle α and the constant ε). The largest deviation of the radial velocity g_j of propagation of the elastic wave for $\varepsilon = 1.025$ from the corresponding velocity g_{0j} for $\varepsilon = 1$ is observed in the $x_1' = 0$ plane at $\alpha = 0$ for the quasitransverse wave propagating with a velocity g_3 and amounts to 2.5%.

A comparative analysis of the velocities g_j and v_j of propagation of elastic waves, which has been performed for ε in the range of 0.975–1.025, shows that their ratio satisfies the inequality $g_j/v_j \geq 1$ irrespective of the angle of inclination of the normal to the characteristic surface.

In closing, we note that formulas (9) can directly be used for determination of the coordinates of the medium's points approached by the wave disturbance by the time t, construction of three-dimensional fronts of elastic waves, and evaluation of the influence of micropolar effects on the angles and positions of lacunas occurring in propagation of quasitransverse waves.

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NOTATION

A_j, elastic constants; A_4, micropolar elastic constant; $A = A_1 - A_2 - A_3 - A_4$; $a = A_1/A_3$; B_j and B_4, micropolar elastic constants; $B = B_1 - B_2 - B_3 - B_4$; $b = A_2/A_3$; $c = \sqrt{A_3/\rho}$; f_j, body forces; G, radial velocity; $g_j = G_j/c$; g_{0j}, radial velocity of propagation of an elastic wave for $\varepsilon = 1$; J, microinertia constant; l_j, body moments; $n_j = p_j/\sqrt{\tau_1}$, direction cosines of the normal to the wave surface; p_0 and p_j, parameters; u_1, u_2, and u_3, components of the displacement vector; $V = -p_0/\sqrt{\tau_1}$, phase velocity; $v = V/c$; v_{0j}, velocity of propagation of an elastic wave for $\varepsilon = 1$; α, angle of inclination of the normal to the wave surface; Δ, Laplace operator; $\varepsilon = A_4/A_3$; ε_{ijm}, alternating tensor; φ_1, φ_2, and φ_3, components of the microrotation vector; ρ, density of the medium; $\tau_1 = p_1^2 + p_2^2 + p_3^2$; $\tau_2 = p_1^2 p_2^2 + p_2^2 p_3^2 + p_1^2 p_3^2$; $\tau_3 = p_1^2 p_2^2 p_3^2$; $\hat{\tau}_3 = n_1^2 n_2^2 n_3^2$; $\hat{\tau}_2 = n_1^2 n_2^2 + n_2^2 n_3^2 + n_1^2 n_3^2$; $\tau_{1i} = 2n_i$; $\tau_{2i} = 2n_i(1 - n_i^2)$; $\tau_{3i} = 2n_i(\hat{\tau}_2 - n_i^2(1 - n_i^2))$; $\partial_j = \partial/\partial x_j$; point, time differentiation. Subscripts: j, i, and $m = 1, 3$.

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